Pre-class Warm-up!!!

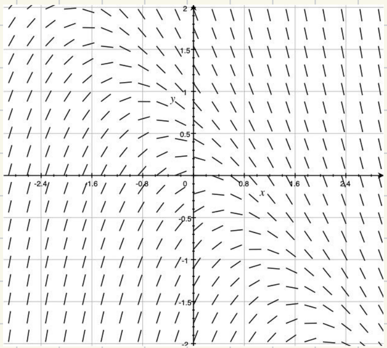
Which slope field is correct for the equation $y' = x + y$?

$x = -y \Rightarrow y' = 0$, $y = 0 \Rightarrow y' \geq 0$ on the positive x axis.

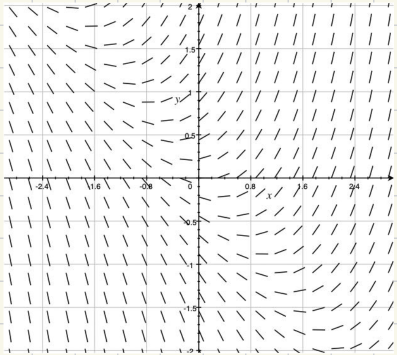
a.
✓ b.
c.
d.
e. None of the above.

The x and y axes are not to the same scale.

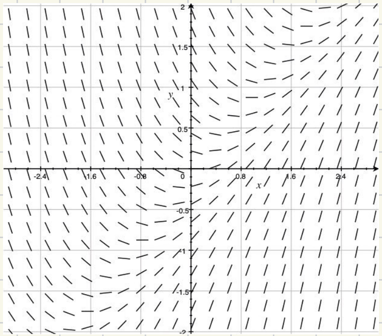
a.



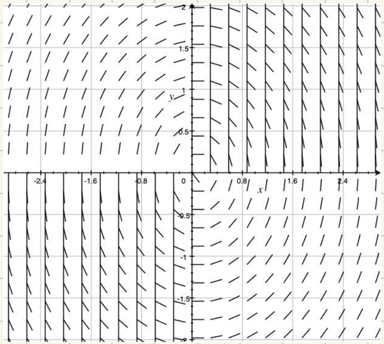
b.



c.



d.



In Sections 1.4, 1.5, 1.6 we learn specific techniques to solve certain types of differential equation:

- 1.4 Separating the variables
- 1.5 Linear first order equations
- 1.6 Special substitutions, homogeneous equations, Bernoulli equations, exact equations, reducing the order of an equation.

1.4 Separating the variables

This applies to differential equations
$dy/dx = g(x)h(y)$
where we can put everything to do with x on one side of the = and everything to do with y on the other side.

We learn

- the method of solving these equations
- It applies to the equations for population growth and Newton's law of cooling.
- Sometimes solutions are given implicitly.
- We meet again the term general solution
- We don't need: singular solutions.

Page 41 question 19:

Solve $\frac{dy}{dx} = ye^x, \quad y(0) = 2e.$

Solution.

$$\int \frac{1}{y}\, dy = \int e^x\, dx$$

$$\ln y = e^x + C \quad \text{for some constant } C$$

$$y = e^{(e^x + C)} = e^{(e^x)} e^C$$

$$= B e^{(e^x)} \quad \text{where } B = e^C$$

Apply the initial condition

$y(0) = 2e$.

Question: in $y = Be^{(e^x)}$ what is the constant B?

a. 1
b. 2 ✓
c. e
d. 3
e. None of the above

Page 41 question 15:
Find the general solution of

$$x^2(2y^2-1)\frac{dy}{dx} = xy^4 - y^4$$

Solution. Note: $xy^4 - y^4 = (x-1)y^4$

$$\int \frac{2y^2-1}{y^4}\,dy = \int \frac{x-1}{x^2}\,dx$$

$$\int \left(\frac{2}{y^2} - \frac{1}{y^4}\right) dy = \int \frac{1}{x} - \frac{1}{x^2}\,dx$$

$$-\frac{2}{y} + \frac{1}{3y^3} = \ln x + \frac{1}{x} + C$$

We can try to make y the subject of the equation, but we can't do it.
y is defined implicitly as a function of x.
Leave it like that!

Done!

Why does the method work?

For an equation $dy/dx = g(x)h(y)$ we get

$$\int \left(\frac{1}{h(y)} \frac{dy}{dx} \right) dx = \int g(x)\, dx$$

$$\parallel$$

$$\int \frac{1}{h(y)}\, dy$$

Which piece of theory did we use to do this?

a. The fundamental theorem of calculus.

b. The chain rule ✓

c. Green's theorem

d. Integration by substitution ✓

e. Something else

Population growth

Page 41 question 34:
In a certain culture of bacteria the number of bacteria increased sixfold in 10 hours. How long did it take for the population to double?

Solution: Let the number of bacteria at time t be $P(t)$. Then

$$\frac{dP}{dt} = kP$$ for some constant k.

Solve the equation:

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln P = kt + C$$ for some constant C.

$$P = e^{kt+C} = Be^{kt}$$ where $B = e^C$

When $t = 10$,

$$P(10) = 6P(0)$$

$$Be^{10k} = 6Be^{0 \cdot k} = 6B$$

$$e^{10k} = 6 \qquad k = \frac{\ln 6}{10}$$

We solve $P(t) = 2P(0)$.

$$Be^{kt} = 2Be^{0k} = 2B$$

$$e^{kt} = 2$$

$$kt = \ln(2)$$

$$t = \frac{1}{k}\ln(2) = \frac{10\ln(2)}{\ln(6)}$$

Newton's law of cooling

Page 42 question 43:
A pitcher of buttermilk initially at 25 degrees C is to be cooled by setting it on the front porch, where the temperature is 0 degrees C. Suppose that the temperature of the buttermilk has dropped to 15 degrees C after 20 min. When will it be at 5 degrees C?

Solution: We use the equation

$$\frac{dT}{dt} = k(T-A)$$

where $T(t)$ is the temperature at time t, A is the ambient temperature. Separate the variables:

$$\int \frac{dT}{T-A} = \int k\,dt$$

$$\ln(T-A) = kt + C$$

$$T - A = e^{kt+C} = Be^{kt}$$

$$T = Be^{kt} + A$$

$$T(0) = 25 = Be^0 + 0, \quad B = 25$$

$$15 = T(20) = 25e^{k20}$$

so $k = \frac{1}{20}\ln\frac{15}{25} = \frac{1}{20}\ln\frac{3}{5}$.

We solve

$$5 = T(t) = 25e^{kt}$$

$$t = \frac{1}{k}\ln\frac{1}{5} = 20\,\frac{-\ln 5}{\ln(3)-\ln(5)}.$$